EXERCISES 1.1

List the elements in each set.

1. The set of natural numbers less than 5.
2. The set of whole numbers greater than 100.
3. The set of whole numbers between 2 and 7.
4. The set of integers greater than \(-3\).
5. The set of negative integers greater than \(-3\).
6. The set of positive integers less than 5.
7. The set of integers less than 1.
8. The set of integers that are not whole numbers.
9. The set of whole numbers that are not integers.
10. The set of integers that are also rational numbers.

Answer true or false to each statement. If false, give a specific counterexample to justify your answer. (Recall that a true statement must be true for all possible cases; otherwise it is false.)

11. The set of whole numbers is closed with respect to multiplication.
12. The set of natural numbers is closed with respect to subtraction.
13. The set of integers is closed with respect to division.
14. Except for 0, the set of rational numbers is closed with respect to division.
15. The set of integers is commutative with respect to subtraction.
16. The set of rational numbers is associative with respect to multiplication.
17. The set of rational numbers contains the additive inverse for each of its members.
18. Except for 0, the set of rational numbers contains the multiplicative inverse for each of its members.
19. The product of any two real numbers is a real number.
20. The quotient of any two real numbers is a real number.

Classify each number as a member of one or more of these sets: (a) natural numbers; (b) whole numbers; (c) integers; (d) rational numbers; (e) irrational numbers; (f) real numbers.

21. \(-15\) 22. 72 23. \(\sqrt{51}\) 24. \(-\frac{1}{3}\) 25. \(\frac{\sqrt{3}}{2}\)
26. 0.01 27. 0 28. 1000 29. \(\sqrt{12}\) 30. \(-\sqrt{2}\)

Name the property illustrated by each of the following.

31. \(5 + 7\) is a real number.
32. \(8 + \sqrt{7} = \sqrt{7} + 8\)
33. \((-5) + 5 = 0\)
34. \(9 + (7 + 6) = (9 + 7) + 6\)
35. \((5 \times 7) \times 8 = (7 \times 5) \times 8\)
36. \((5 \times 7) \times 8 = 5 \times (7 \times 8)\)
37. \(-\frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}\)
38. \((4 \times 5) + (4 \times 8) = 4(5 + 8)\)
39. \(-13 + 0 = -13\)
40. \(1 \times \frac{1}{2} = \frac{1}{2}\)
41. \(\frac{1}{2} + (-\frac{1}{2}) = 0\)
42. \(3 - 7 = 3 + (-7)\)
43. \(0(\sqrt{2} + \sqrt{3}) = 0\)
44. \(\sqrt{2} \times \pi \) is a real number.
45. \((3 + 9)(7) = (3)(7) + (9)(7)\)
46. \(\frac{1}{\sqrt{2}} \times \sqrt{2} = 1\)

Replace the variable \(n\) by a real number to make each statement true.

47. \(7 + n = 3 + 7\)
48. \(\sqrt{5} \times 6 = 6 \times n\)
49. \((3 + 7) + n = 3 + (7 + 5)\)
50. \(6 \times (5 \times 4) = (6 \times n) \times 4\)
51. \((5\sqrt{8} + n) = (5 \times 8) + (5 \times 7)\)
52. \((3 \times 7) + (3 \times n) = 3(7 + 5)\)
EXERCISES 1.2

Solve for x and check each result.

1. \(3x - 2 = 10\) 
2. \(5x + 1 = 21\) 
3. \(-2x + 1 = 9\) 
4. \(-3x - 2 = 10\) 
5. \(-3x - 5 = 7\) 
6. \(3x + 2 = -13\) 
7. \(2x - 1 = -17\) 
8. \(-2x + 3 = -12\) 
9. \(2(x + 1) = 11\) 
10. \(3(x - 2) = 15\) 
11. \(3x + 7 = 2x - 2\) 
12. \(2.5x - 8 = x + 3\) 
13. \(\frac{1}{2}x + 7 = 2x - 3\) 
14. \(\frac{1}{2}x - 5 = 3x + 7\) 
15. \(\frac{1}{3}x - 7 = \frac{1}{3}x + 8\) 
16. \(5x - 1 = 5x + 1\) 
17. \(\frac{3}{2}(x - 5) = x + 1\) 
18. \(5(x + 4) = \frac{5}{2}x - 5\) 
19. \(\frac{3}{4}x + 5 + \frac{1}{4}x = \frac{3}{2}x - 6\) 
20. \(2(x + 3) - x = 2x + 8\) 
21. \(-3(x + 2) + 1 = x - 25\) 
22. \(\frac{1}{2}(x + 8) = \frac{1}{2}(2x + 12)\) 
23. \(1 - 12x = 7(1 - 2x)\) 
24. \(2(3x - 7) - 4x = -2\) 
25. \(x + 2(\frac{1}{2}x + 2) = \frac{3}{2}x + 16\)

Solve for the indicated variable.

26. \(P = 4s\) for \(s\)
27. \(P = 2l + 2w\) for \(w\)
28. \(F = \frac{2}{3}C + 32\) for \(C\)
29. \(N = 10t + u\) for \(t\)
30. \(7a - 3b = c\) for \(b\)
31. \(C = 2\pi r\) for \(r\)
32. \(2(r - 3s) = 6t\) for \(s\)
33. \(6 + 4v = w - 1\) for \(v\)

34. Find a number such that two-thirds of the number increased by one is 13.
35. Find the dimensions of a rectangle whose perimeter is 56 inches if the length is three times the width.
36. Each of the two equal sides of an isosceles triangle is 3 inches longer than the base of the triangle. The perimeter is 21 inches. Find the length of each side.
37. Carlos spent $4.85 on stamps, in denominations of 10¢, 20¢, and 25¢. He bought one-half as many 25¢ stamps as 10¢ stamps, and three more 20¢ stamps than 10¢ stamps. How many of each type did he buy?
38. Maria has $169 in ones, fives, and tens. She has twice as many one-dollar bills as she has five-dollar bills, and five more ten-dollar bills than five-dollar bills. How many of each type bill does she have?
39. Two cars leave a town at the same time and travel in opposite directions. One car travels at the rate of 45 miles per hour, and the other at 55 miles per hour. In how many hours will the two cars be 350 miles apart?
40. Robert goes for a walk at a speed of 3 miles per hour. Two hours later Roger attempts to overtake him by jogging at the rate of 7 miles per hour. How long will it take him to reach Robert?
41. Prove that the measures of the angles of a triangle cannot be represented by consecutive odd integers. (Hint: The sum of the measures is 180.)
42. The width of a painting is 4 inches less than the length. The frame that surrounds the painting is 2 inches wide and has an area of 240 square inches. What are the dimensions of the painting? (Hint: The total area minus the area of the painting alone is equal to the area of the frame.)
43. The length of a rectangle is 1 inch less than three times the width. If the length is increased by 6 inches and the width is increased by 5 inches, then the length will be twice the width. Find the dimensions of the rectangle.
44. The units' digit of a two-digit number is three more than the tens' digit. The number is equal to four times the sum of the digits. Find the number. (Hint: We can represent a two-digit number as \(10t + u\).)
45. Find three consecutive odd integers such that their sum is 237. (Hint: The three integers can be represented as \(x, x + 2, \text{ and } x + 4\).)
46. The length of a rectangle is 1 inch less than twice the width. If the length is increased by
11 inches and the width is increased by 5 inches, then the length will be twice the width. What can you conclude about the data for this problem?

47. Amy travels 27.5 miles to get to work by car. The first part of her trip is along a country road on which she averages 35 miles per hour, and the second part is on a highway where she averages 48 miles per hour. If the time she travels on the highway is 5 times the amount of time she travels on the country road, what is the total time for the trip?

48. A taxi charges 80¢ for the first ½ mile and 20¢ for each additional ½ mile. If a passenger paid $6.00, how far did the taxi travel?

49. A financial advisor invested an amount of money at an annual rate of interest of 9%. She invested $2700 more than this amount at 12% annually. The total yearly income from these investments was $1794. How much did she invest at each rate? (Note: Use the formula \( I = Prt \), where \( I \) is the interest earned on the principal of \( P \) dollars invested at the rate \( r \) (in decimal form) per year. In this case the time, \( t \), is 1 year.)

50. Luis earns a monthly salary of $1225, plus a commission of 8% of his total sales for the month. Last month his total earnings were $1750. What were his total sales?

51. Leslie paid $9010 for a used car, which included a 6% sales tax on the base cost of the car. What was the cost of the car, without the sales tax?

52. The total cost of two certificates of deposit is $12,800. The annual interest rates of these certificates are 8% and 9%. The yearly interest on the 9% certificate is $217 more than that on the 8% certificate. What is the cost of each certificate?

53. Following is a set of directions for a mathematical trick. First try it. Then use algebraic representations for each phrase (direction) to show why the trick works.

Think of a number.
Add 2.
Multiply by 3.
Add 9.
Multiply by 2.
Divide by 6.
Subtract the number with which you started.
The result is 5.

CHALLENGE
Think Creatively

Two boats begin their journeys back and forth across a river at the same time, but from opposite sides of the river. The first time that they pass each other they are 700 feet from one of the shores of the river. After they each make one turn, they pass each other once again at a distance of 400 feet from the other shore. How wide is the river? Assume each boat travels at a constant speed and that there is no loss of time in making a turn.

1.3
STATEMENTS
OF INEQUALITY
AND THEIR
GRAPHS

As you continue your study of mathematics you will find a great deal of attention given to inequalities. We begin our discussion of this topic by considering the ordering of the real numbers on the number line. In the following figure we say that \( a \) is less than \( b \) because \( a \) lies to the left of \( b \). In symbols, we write \( a < b \).

\[ a \quad b \]

Also note that \( b \) lies to the right of \( a \). That is, \( b > a \); this is read "\( b \) is greater than \( a \)." Two inequalities, one using the symbol < and the other >, are said to have the opposite sense.